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3-Modulo Cordial Labeling of Some Graphs

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Abstract

Let $G = (V, E)$ be a simple graph with p vertices and q edges. G is said to admit 3-modulo cordial labeling if there is a injective map $f: V(G) \rightarrow \{0,1,2,3, \dots, 3p\}$ such that for every edge uv, the induced edge labeling f^* defined as $f^*(uv) = 1$ if $f(u) + f(v) \equiv 0(mod\ 3)$ and 0 elsewhere satisfies the condition that $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1. If G admits 3-modulo cordial labeling then G is called a 3-modulo cordial graph. In this paper, we analyze some graphs for 3-modulo cordial labeling.

Keywords: Cordial, Cordial labeling, 3-Modulo Cordial Graph.
AMS Subject Classification: 05C78.

I.Introduction

The concept of cordial labeling is introduced by Cahit in [1] and proved certain results in cordial labeling in [2].The concept of 3-modulo cordial graph was introduced by A.Nagarajan et.al[4].ForP_n^2, open ladder,triangular ladder and closed helm refer [3]. In this paper, we analyze some graphs for 3-modulo cordial labeling.

II. Main Results

2.1 Defintion: Let $G = (V, E)$ be a simple graph with p vertices and q edges. G is said to admit 3-modulo cordial labeling if there is a injective map $f: V(G) \rightarrow \{0,1,2,3, \dots, 3p\}$ such that for every edge uv, the induced edge labeling f^* defined as $f^*(uv) = 1$ if $f(u) + f(v) \equiv 0(mod\ 3)$ and 0 elsewhere satisfies the condition that $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1. If G admits 3-modulo cordial labeling then G is called a 3-modulo cordial graph.

2.2 Theorem: $P_n^2$is a 3 – modulo cordial graph.

Proof:

Let G be a graph

Let V(G) = $\{ u_1, u_2, u_3, \dots \dots \dots u_n\}$

E(G) = $\{ u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i u_{i+2} / 1 \leq i \leq n-2\}$
Then $|V(G)| = n$ and $|E(G)| = 2n - 3$
Define $f: V(G) \rightarrow \{0,1,2,3, \dots \dots \dots ., 3n\}$
Case 1: Suppose n is odd , say n = 2k + 1
The vertex labels are,

$$f(u_i) = \begin{cases} 3i-1 , & 1 \le i \le k \\ 3i , & k+1 \le i \le n \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-1$

$f^*(u_i u_{i+1}) = 6i + 1 \equiv 1 (mod\ 3)$

For $k+1 \le i \le n-1$

$f^*(u_i u_{i+1}) = 6i + 3 \equiv 0 (mod\ 3)$

$f^*(u_k u_{k+1}) = 6k + 2 \equiv 2 (mod\ 3)$

For $1 \le i \le k-2$

$f^*(u_i u_{i+2}) = 6(i+1) - 2 \equiv 1 (mod\ 3)$

For $k+1 \le i \le n-2$

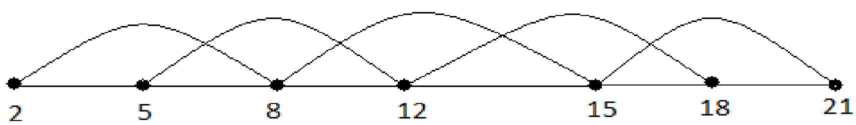
$f^*(u_i u_{i+2}) = 6(i+1) \equiv 0 (mod\ 3)$

$f^*(u_{k-1} u_{k+1}) = 6k - 1 \equiv 2 (mod\ 3)$

$f^*(u_k u_{k+2}) = 6k + 5 \equiv 2 (mod\ 3)$

It is observed as

$e_f(0) = 2k \& e_f(1) = 2k - 1$



$Fig. 2.1$

Case 2: Suppose n is even , say n = 2k

The vertex labels are,

$$f(u_i) = \begin{cases} 3i-1 , & 1 \le i \le k-1 \\ 3i , & k \le i \le n \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-2$

$f^*(u_i u_{i+1}) = 6i + 1 \equiv 1 (mod\ 3)$

For $k \le i \le n-1$

$f^*(u_i u_{i+1}) = 6i + 3 \equiv 0 (mod\ 3)$

$f^*(u_{k-1} u_k) = 6k - 4 \equiv 2 (mod\ 3)$

For $1 \le i \le k-3$

$f^*(u_i u_{i+2}) = 6i + 4 \equiv 1 (mod\ 3)$

For $k \le i \le n-2$

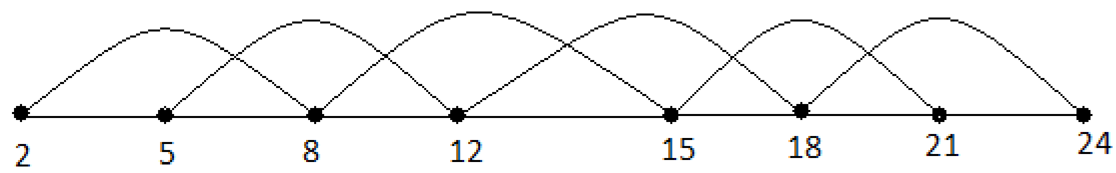
$f^*(u_i u_{i+2}) = 6(i+1) \equiv 0 (mod\ 3)$

$f^*(u_{k-2} u_k) = 6(k-1) - 1 \equiv 2 (mod\ 3)$

$f^*(u_{k-1} u_{k+1}) = 6k - 1 \equiv 2 (mod\ 3)$

It is observed as

$e_f(0) = 2k - 2 \& e_f(1) = 2k - 1$



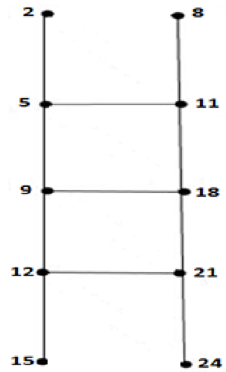
$Fig. 2.2$

Clearly $|e_f(0) - e_f(1)| \leq 1$
Then f is a 3 – modulo cordial labeling.
Hence P_n^2 is a 3 – modulo cordial graph.
2.3Theorem: The Open Ladder $O(L_n)$ is a 3 – modulo cordial graph.
Proof:
Let G be a graph
Let V(G) = $\{ u_1, u_2, u_3, \dots \dots \dots u_n, v_1, v_2, v_3, \dots \dots \dots v_n\}$
E(G) = $\{ u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{ v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_i / 2 \leq i \leq n-1\}$
Then $|V(G)| = 2n$ and $|E(G)| = 3n - 3$
Define $f: V(G) \to \{0,1,2,3, \dots \dots \dots, 6n\}$
Case 1: Suppose n is odd , say n = 2k + 1
The vertex labels are,

$$f(u_i) = \begin{cases} 3i - 1 & , \quad 1 \leq i \leq k \\ 3i & , \quad k+1 \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} 3(k+i) - 1 & , \quad 1 \leq i \leq k \\ 3(k+i) + 3 & , \quad k+1 \leq i \leq n \end{cases}$$

The induced edge labels are,
For $1 \leq i \leq k$
$f^*(u_i u_{i+1}) = 6i + 1 \equiv 1 (mod\ 3)$
For $k + 1 \leq i \leq n - 1$
$f^*(u_i u_{i+1}) = 6i + 3 \equiv 0 (mod\ 3)$
$f^*(u_k u_{k+1}) = 6k + 2 \equiv 2 (mod\ 3)$
For $1 \leq i \leq k$
$f^*(v_i v_{i+1}) = 6(k + i) + 1 \equiv 1 (mod\ 3)$
For $k + 1 \leq i \leq n - 1$
$f^*(v_i v_{i+1}) = 6k + 6i + 9 \equiv 0 (mod\ 3)$
$f^*(v_k v_{k+1}) = 12k + 5 \equiv 2 (mod\ 3)$
For $2 \leq i \leq k$
$f^*(u_i v_i) = 3k + 6i - 2 \equiv 1 (mod\ 3)$
For $k + 1 \leq i \leq n - 1$
$f^*(u_i v_i) = 3k + 6i + 3 \equiv 0 (mod\ 3)$
It is observed as
$e_f(0) = 3k$-1&$e_f(1) = 3k$



$Fig. 2.3$

Case 2: Suppose n is even , say n = 2k

The vertex labels are,

$f(u_i) = \begin{cases} 3i-1 & , \quad 1 \le i \le k \\ 3i & , \quad k+1 \le i \le n \end{cases}$

$f(v_i) = \begin{cases} 3(k+i)-1 & , \quad 1 \le i \le k-1 \\ 3(k+i)+3 & , \quad k \le i \le n \end{cases}$

The induced edge labels are,

For $1 \le i \le k-1$

$f^*(u_i u_{i+1}) = 6i + 1 \equiv 1(mod\ 3)$

For $k+1 \le i \le n-1$

$f^*(u_i u_{i+1}) = 6i + 3 \equiv 0(mod\ 3)$

$f^*(u_k u_{k+1}) = 6k + 2 \equiv 2(mod\ 3)$

For $1 \le i \le k-2$

$f^*(v_i v_{i+1}) = 6(k+i) + 1 \equiv 1(mod\ 3)$

For $k \le i \le n-1$

$f^*(v_i v_{i+1}) = 6k + 6i + 9 \equiv 0(mod\ 3)$

$f^*(v_{k-1} v_k) = 12k - 1 \equiv 2(mod\ 3)$

For $2 \le i \le k-1$

$f^*(u_i v_i) = 3k + 6i - 2 \equiv 1(mod\ 3)$

For $k+1 \le i \le n-1$

$f^*(u_i v_i) = 3k + 6i + 3 \equiv 0(mod\ 3)$

$f^*(u_k v_k) = 9k + 2 \equiv 2(mod\ 3)$

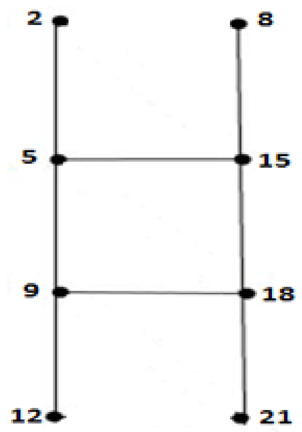
It is observed as

$e_f(0) = 3k - 2 \& e_f(1) = 3k - 2$

Clearly $|e_f(0) - e_f(1)| \le 1$

Then f is a 3 – modulo cordial labeling.

Hence $O(L_n)$ is a 3 – modulo cordial graph.



$Fig. 2.4$

2.4 Theorem: The Triangular ladder(TL_n) is a 3 – modulo cordial graph.

Proof:

Let G be a graph

Let V(G) = $\{ u_1, u_2, u_3, \dots \dots \dots u_n, v_1, v_2, v_3, \dots \dots \dots v_n\}$

E(G) = $\{ u_i u_{i+1} / 1 \le i \le n-1\} \cup \{ v_i v_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i / 1 \le i \le n\} \cup \{ u_i v_{i+1} / 1 \le i \le n-1\}$

Then $|V(G)| = 2n$ and $|E(G)| = 4n-3$

Define $f: V(G) \to \{0,1,2,3, \dots \dots \dots, 6n\}$

Case 1: Suppose n is odd , say n = 2k + 1

The vertex labels are,

$$f(u_i) = \begin{cases} 3i-1 & , \quad 1 \le i \le k \\ 3i & , \quad k+1 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} 3(k+i)-1 & , \quad 1 \le i \le k \\ 3(k+i)+3 & , \quad k+1 \le i \le n \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-1$

$f^*(u_i u_{i+1}) = 6i + 1 \equiv 1 (mod\ 3)$

For $k+1 \le i \le n-1$

$f^*(u_i u_{i+1}) = 6i + 3 \equiv 0 (mod\ 3)$

$f^*(u_k u_{k+1}) = 6k + 2 \equiv 2 (mod\ 3)$

For $1 \le i \le k-1$

$f^*(v_i v_{i+1}) = 6(k+i) + 1 \equiv 1 (mod\ 3)$

For $k+1 \le i \le n-1$

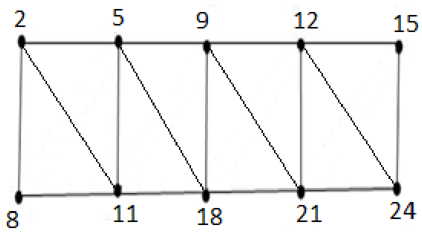
$f^*(v_i v_{i+1}) = 6k + 6i + 9 \equiv 0 (mod\ 3)$

$f^*(v_k v_{k+1}) = 12k + 5 \equiv 2 (mod\ 3)$

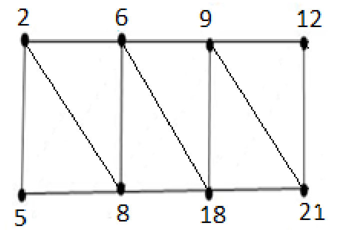
For $1 \le i \le k$

$f^*(u_i v_i) = 3k + 6i - 2 \equiv 1 (mod\ 3)$

For $k+1 \leq i \leq n$
$f^*(u_i v_i) = 3k + 6i + 3 \equiv 0(mod\ 3)$
For $1 \leq i \leq k-1$
$f^*(u_i v_{i+1}) = 3k + 6i + 1 \equiv 1(mod\ 3)$
For $k+1 \leq i \leq n-1$
$f^*(u_i v_{i+1}) = 3k + 6i + 6 \equiv 0(mod\ 3)$
$f^*(u_k v_{k+1}) = 9k + 5 \equiv 2(mod\ 3)$
It is observed as
$e_f(0) = 4k \& e_f(1) = 4k + 1$
Case 2: Suppose n is even , say n = 2k
The vertex labels are,
$$f(u_i) = \begin{cases} 3i - 1 & , \quad 1 \leq i \leq k-1 \\ 3i & , \quad k \leq i \leq n \end{cases}$$
$$f(v_i) = \begin{cases} 3(k+i) - 4 & , \quad 1 \leq i \leq k \\ 3(k+i) & , \quad k+1 \leq i \leq n \end{cases}$$
The induced edge labels are,
For $1 \leq i \leq k-2$
$f^*(u_i u_{i+1}) = 6i + 1 \equiv 1(mod\ 3)$
For $k \leq i \leq n-1$
$f^*(u_i u_{i+1}) = 6i + 3 \equiv 0(mod\ 3)$
$f^*(u_{k-1} u_k) = 6k - 4 \equiv 2(mod\ 3)$
For $1 \leq i \leq k-1$
$f^*(v_i v_{i+1}) = 6(k+i) + 1 \equiv 1(mod\ 3)$
For $k+1 \leq i \leq n-1$
$f^*(v_i v_{i+1}) = 6k + 6i + 3 \equiv 0(mod\ 3)$
$f^*(v_k v_{k+1}) = 12k - 1 \equiv 2(mod\ 3)$
For $1 \leq i \leq k-1$
$f^*(u_i v_i) = 3k + 6i - 5 \equiv 1(mod\ 3)$
For $k+1 \leq i \leq n$
$f^*(u_i v_i) = 3k + 6i \equiv 0(mod\ 3)$
$f^*(u_k v_k) = 9k - 4 \equiv 2(mod\ 3)$
For $1 \leq i \leq k-1$
$f^*(u_i v_{i+1}) = 3k + 6i - 2 \equiv 1(mod\ 3)$
For $k+1 \leq i \leq n-1$
$f^*(u_i v_{i+1}) = 3k + 6i + 3 \equiv 0(mod\ 3)$
$f^*(u_k v_{k+1}) = 9k + 3 \equiv 0(mod\ 3)$
It is observed as
$e_f(0) = 4k - 2 \& e_f(1) = 4k - 1$
Clearly $|e_f(0) - e_f(1)| \leq 1$
Then f is a 3 – modulo cordial labeling.
Hence TL_n is a 3 – modulo cordial graph.



$Fig. 2.5$



$Fig. 2.6$

2.5 Theorem: The Closed Helm CH_n is a 3 – modulo cordial graph.
Proof:
Let G be a graph
Let V(G) = { $u_1, u_2, u_3, \dots \dots \dots u_n, v_1, v_2, v_3, \dots \dots \dots v_n, v$}
E(G) = $\{u_i u_{i+1}/1 \leq i \leq n-1\} \cup \{u_i v_i/1 \leq i \leq n\} \cup \{u_n u_1\} \cup \{u_i v/1 \leq i \leq n\} \cup$
$\{v_i v_{i+1}/1 \leq i \leq n-1\} \cup \{v_n v_1\}$
Then $|V(G)| = 2n + 1$ and $|E(G)| = 4n$
Define $f: V(G) \rightarrow \{0,1,2,3, \dots \dots \dots ., 6n + 3\}$
Case 1: Suppose n is odd , say n = 2k + 1
The vertex labels are,
$f(u_i) = 3i \quad , \quad 1 \leq i \leq n$
$$f(v_i) = \begin{cases} 3i - 1 & , \quad 1 \leq i \leq k \\ 3(k + i + 1) & , \quad k + 1 \leq i \leq n \end{cases}$$
$f(v) = 1$
The induced edge labels are,
For 1≤ $i \leq n - 1$
$f^*(u_i u_{i+1}) = 3(2i + 1) \equiv 0(mod\ 3)$
$f^*(u_n u_1) = 3n + 3 \equiv 0(mod\ 3)$
For 1≤ $i \leq n$
$f^*(u_i v) = 3i + 1 \equiv 1(mod\ 3)$
For 1≤ $i \leq k - 1$
$f^*(v_i v_{i+1}) = 6i + 1 \equiv 1(mod\ 3)$
For $k + 1 \leq i \leq n - 1$
$f^*(v_i v_{i+1}) = 6(k + i) + 9 \equiv 1(mod\ 3)$
$f^*(v_n v_1) = 3(k + n) + 5 \equiv 2(mod\ 3)$
$f^*(v_k v_{k+1}) = 9k + 5 \equiv 2(mod\ 3)$
For 1≤ $i \leq k$
$f^*(u_i v_i) = 6i - 1 \equiv 2(mod\ 3)$
For $k + 1 \leq i \leq n$
$f^*(u_i v_i) = 3(k + 2i + 1) \equiv 0(mod\ 3)$
It is observed as
$e_f(0) = 4k + 2 \& e_f(1) = 4k + 2$

Case 2: Suppose n is even , say n = 2k .

The vertex labels are,

$f(u_i) = 3i \quad , \quad 1 \leq i \leq n$

$f(v_i) = 3i - 1 \quad , \quad 1 \leq i \leq n$

$f(v) = 0$

The induced edge labels are,

For 1$\leq i \leq n-1$

$f^*(u_i u_{i+1}) = 6i + 3 \equiv 0 (mod\ 3)$

$f^*(u_n u_1) = 3n + 3 \equiv 0 (mod\ 3)$

For 1$\leq i \leq n$

$f^*(u_i v) = 3i \equiv 0 (mod\ 3)$

For 1$\leq i \leq n-1$

$f^*(v_i v_{i+1}) = 6i + 1 \equiv 1 (mod\ 3)$

$f^*(v_n v_1) = 3n + 1 \equiv 1 (mod\ 3)$

For 1$\leq i \leq n$

$f^*(u_i v_i) = 6i - 1 \equiv 2 (mod\ 3)$

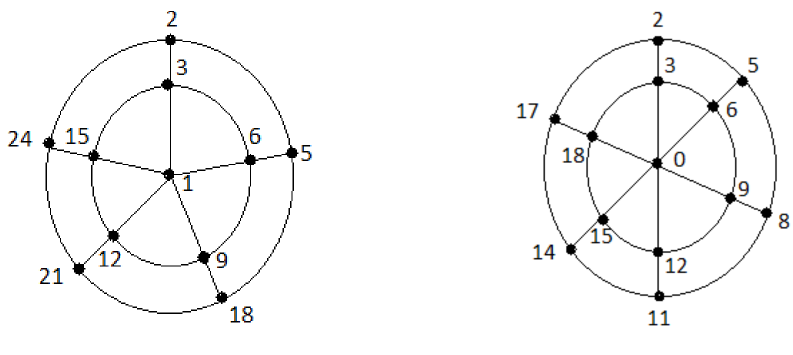
It is observed as

$e_f(0) = 4k \& e_f(1) = 4k$

Clearly $|e_f(0) - e_f(1)| \leq 1$

Then f is a 3 – modulo cordial labeling.

Hence $CH_n$is a 3 – modulo cordial graph.



$Fig. 2.7$ $\qquad$ $Fig\ 2.8$

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